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ABSTRACT

A mathematics curriculum for moderately and severely retarded and autistic students and adults should possess 10 necessary characteristics, including provision for instruction based on assessment results, stress on functional application, focus on generalization of skills, and specific criteria for mastery of each skill. The functional but developmentally based program described was organized around 47 skills in two sections: the perceptual correspondence and the numerical correspondence curricula. Perceptual correspondence deals with the student's ability to understand separate objects (discontinuous quantities) and includes concepts of "one/many" and "more." Numerical correspondence covers the development of qualitative and numerical seriation and includes functional addition, subtraction, multiplication, and division. Language related concerns in the development of the curriculum included students' tendencies to answer affirmatively to yes or no questions, and difficulties with understanding the words "one," "amount," and "enough." Sequences of instruction were coordinated with theories of J. Piaget. Specific teaching techniques for each skill level were developed using generalized materials and examples. Differences were noted between autistic and retarded students in performing mathematics curriculum tasks. The curriculum has also been effective in teaching functional mathematics to elementary students with mild retardation and learning disabilities. (CL)

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TEACHING BEGINNING MATHEMATICS TO STUDENTS WITH RETARDATION, AUTISM, AND LEARNING DISABILITIES

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TEACHING BEGINNING MATHEMATICS TO STUDENTS WITH RETARDATION, AUTISM, AND LEARNING DISABILITIES

When the term "mathematics" is mentioned, people commonly think of numbers and numerical operations--counting, long division, algebra--and possibly of problem-solving as required for "story problems." However, basic mathematics skills are a part of most cognitive tasks. Whether a person takes only one item of food, sorts (set recognition), packages by matching to a model or using outlines on a jib, fills a cup full, or waits ten minutes, that person is functionally using basic math skills.

Piaget carefully observed and recorded the order of development of basic cognitive processes, and decades of experimentation and replication have proven him essentially correct. However, our exhaustive search of published mathematics assessments and curriculum typically showed a sequence of skills or objectives outlining rote counting first to five, then to ten, and so on, followed by numeral recognition, writing numerals, and simple addition. The result is a math curriculum which is logically correct but developmentally unsound. Students with little or no expressive language cannot be included, although the developmental beginnings of mathematics do not require expressive language. Many students do not experience success, and for those who do, a frequent result is a student who can add 3 + 4 when it is written in a form with which he is familiar but who cannot figure out how many glasses are needed so that everyone in the room can have a glass of punch. The student has learned "splinter skills" beyond his level of cognitive development. These skills, often rote counting and numeral recognition, are nonfunctional for his ability level and environment. Therefore, he cannot generalize them, and they have taught him nothing about problem-solving.

Our original focus was to develop a functional but developmentally-based mathematics curriculum for moderately and severely retarded and autistic students and adults. To our delight, we have found our curriculum to also be very effective with four to nine year old students who are mildly retarded, learning disabled, or simply having difficulty with the arithmetic being mastered by the rest of the class.

This paper will include the following:

1. A description of the necessary ten characteristics of a mathematics curriculum for moderately and severely retarded and autistic students,
2. An explanation of the relevant differences between retarded and non-retarded persons of the same cognitive level,
3. A description of the mathematics program and how it was developed from our interactions with 72 moderately and severely retarded and autistic students,
4. A qualitative description of differences between autistic students and students of approximately equal overall functioning level who are designated retarded but not autistic,
5. A qualitative description of use of the program with young students who are learning disabled or mildly retarded.

Studies of validation of effectiveness and a more detailed discussion of the particular problems of teaching these skills to students who have autism can be found in an article in The Journal of Special Education Technology (in press).

A mathematics curriculum for moderately and severely retarded and autistic students should have 10 characteristics:

1. It must be based on an assessment which provides clear and specific instructions for administration. Only by determining exactly what a student does and does not know can one avoid boring him (by teaching well below his level) or frustrating him (by teaching beyond the level at which he is currently functioning).
2. This assessment should of course be in correct cognitive developmental order, but in addition, it must include all skills important to conceptual development. A common failure of assessment instruments is to test those skills easy to test and which provide age-equivalent landmarks while excluding essential conceptual skills.

3. The skills assessed must progress in steps so small that they will pinpoint differences in current conceptual functioning and indicate the next skill which the student can be expected to master even for very severely retarded persons.

4. Assumptions about receptive and expressive language ability reflected in the assessment must not exceed the conceptual level of the skill being expressed and might best recognize the relative language deficiency of retarded persons compared to their conceptual level.

5. The curriculum must provide instructions for teaching based on the results of the assessment. Skilled teachers may always substitute their own approaches, but the provision of teaching instructions makes regular and correct instruction of students more likely in the hands of harried teachers, of paraprofessionals, and of personnel in non-school settings such as sheltered workshops.

6. Teaching procedures must stress functional application. The needs of the student who is mentally retarded or autistic are different from the needs of the non-handicapped student. The latter learns elementary mathematics as a base to learning more complex mathematics. The early elementary school mathematics curriculum is not considered an end in itself but the base for learning computation skills necessary for long division, for fractions, for decimals, for bookkeeping for some, elementary algebra for others. Further, simple story problems (requiring reading ability) lead to the use of mathematics for solving problems. Students who are retarded or autistic do not need to learn a base for more complex skills. They need rather to learn functional mathematics from the beginning. Functional skills, lacking in traditional mathematics programs, must be built into every step of the procedure. Not only is the teaching of functional mathematics skills a more justifiable use of teaching time, but use of such skills is reinforcing to both students and teachers and will improve skill retention.

7. Generalization of the skill must be built into the teaching procedure. Brown, Nietupski, and Hamre-Nietupski (1976) suggest: "Each time a student is taught a functional skill or game, it might be required that the skill be performed:
   a) in reaction to, or in the presence of at least three different persons;
   b) in at least three different natural settings;
   c) in response to at least three different sets of instructional materials;
   d) to at least three different appropriate language cues." (p. 10)

8. Teaching procedures must include teaching of strategies for problem-solving. This interfaces with both generalization and with functional use of skills. If the student is dependent on a problem being set up in a specific way in order for him to be able to solve it, he has not really mastered that skill.

9. Criteria for mastery for each skill must be specific and must be based on considerations of functional use, generalization, and problem-solving.

10. Materials and methods must be age-appropriate for adolescents and adults. Pictures of clowns and puppies and valentine hearts as cognitive curriculum materials send a message of childhood, not of functional adulthood. On the other hand, younger children as well as adolescents and adults respond positively to kitchen utensils, coins, and workshop materials.

In the course of our own observations of retarded persons, we found three behavioral differences between retarded and non-retarded persons of the same cognitive level which affected our program. (It is not the purpose of this paper to discuss whether the differences are neurological, based in chronological age difference alone, or due to differences in environment and expectations level.) First (and obviously), because the moderately or severely retarded person goes through each stage so much more slowly, he is considerably older than the non-handicapped student at the same cognitive level. Second are language-specific considerations in assessment and teaching which will be discussed in later sections. Third, we observed at many cognitive levels what we call the "no stop syndrome," the tendency to continue until all items are used. Cognitively, this is measured very early as the second skill of the program. The student mastering the ability to match two sets on a one-to-one basis where there are unequal amounts is still in the sensori-motor stage. Yet we saw this tendency long after the skill was mastered. Students who had the skill of object counting would bring all objects when a specific number was requested. Yet when asked how many were wanted, the student would state the correct amount, and the student had previously counted that amount. The same behavior was observed when the student was asked to count a certain number of coins into a container. Piagetian theory doesn't address this particular problem.

Our best guess is that our students were carefully taught to finish tasks, to use everything, i.e., put all the pegs in the pegboard, sort all the color cards, put items together until you've used all the items in front of you. Because of this tendency and because of the importance of perseverance to completion, we included both skills requiring completion and skills requiring matching of sets or counting a certain amount and then stopping. Our students thus learned to differentiate the two types of directions.

The relatively poor language skills of our retarded and autistic subjects prompted the major organization of the curriculum. The 47 skills included are divided into two sections. The first, Perceptual Correspondence Curriculum, requires no expressive language or counting ability, while the second, Numerical Correspondence Curriculum, requires verbal, sign, picture board, or electronic language ability. While the Perceptual Correspondence Curriculum begins with cognitive skills corresponding to the latter part of the sensori-motor period, early preoperational skills are also included and have been designed so as not to require expressive language.

The Perceptual Correspondence and Numerical Correspondence Curriculums each include a developmentally-based, hierarchical assessment to determine skill level. In addition to specific teaching instructions for each of these skill levels, functional skills are described which can be performed by the person functioning at that cognitive level (or a lower level). In some cases environmental manipulation is necessary for this purpose. For example, the fifth skill level in the Perceptual Correspondence Curriculum is a one-to-one

match of each object to its outline. The student who masters this skill would be able to set a table if his parent or teacher provided placemats with outlines of plates and utensils in their proper place drawn on each placemat. Using the same principle, complicated packaging jobs could be carried out by a very severely retarded person in a workshop setting. These ideas for functional skills are not new. What is new is an assessment enabling teachers and others to know exactly when a student has the necessary cognitive development to master the functional skill. Descriptions of each skill level and examples of functional skills at each level are provided so that the practitioner can identify functional skills appropriate to the student's school, living, or work environment as well as appropriate to her cognitive level. In addition to functional skills, games have been devised for each skill level for additional practice, speed, accuracy, generalization, and fun.

The Perceptual Correspondence Curriculum is composed of 16 levels which analyze the ability of the student to deal with discontinuous quantities (separate objects) and then teach the skills not yet attained. It begins with object identity, i.e., the student's ability to perceive object individuality or "oneness," and the beginning of purposeful object placement. By investigating the hierarchy of object placement (both grouping and alignment), we have determined fine discriminations in development levels. Through the provision of the appropriate structure and a hierarchy of perceptual cueing systems, the student is able to perform and learn skills and generalize to functional tasks which were not previously within his repertoire. These skills include the placing of objects into sets under a variety of conditions,

constructing identical sets by imitation from a model, discerning and excluding extra items, completion of partial sets, and construction of remembered sets. At the upper end of the Perceptual Correspondence Curriculum, the student must distinguish between or create two differing amounts, making receptive language-based judgments about the concepts of "one/many" and "more." (The concept of "less" develops years later, even in non-handicapped children.) At this point, the student is firmly into the preoperational stage of dealing with language symbols and making perceptual comparisons.

The Numerical Correspondence Curriculum, composed of 31 skill levels, deals with the development of seriation, both qualitative and numerical, includes functional addition, subtraction, multiplication, and division, concludes with conservation of amount (the first type of conservation to develop) and the concept of "less." We found many students considered capable of counting whose skills in this area were limited to counting objects lined up in a row. For example, when a more functional task was presented, such as counting objects placed in a heap or a pile, common responses included counting only the objects on top or spreading them out without any systematic approach to organization which resulted in objects being counted more than once. Another example of a functional but complicated counting skill that is needed is to correctly add three cans of water to make frozen juice. At a lower cognitive level, having three empty cans available and teaching the person to fill all three

with water, then empty all three into the juice container, makes this possible. However, when the student reaches Skill Level 7 on the Numerical Correspondence Curriculum, he is ready to count and remember as each container of water "disappears" into the juice container and can make juice without special aids. It is important to know when to provide aids and when they are no longer necessary. The curriculum thus includes a carefully developed hierarchy of counting skills inclusive of those needed for everyday functioning.

A functional approach to the arithmetic operations (addition, subtraction, multiplication, division) comprise a number of the skill levels. Our investigation showed that paper-and-pencil arithmetic was based on rote learning, not on conceptual development. The skills must develop in their hierarchical order from action-based experiential teaching and opportunities. Then, some students will be able to use paper-and-pencil to extend their functional ability to larger amounts.

This is also true for the stimulus-response mode of matching the name of the numerals to their written form. Many students could name written numerals or identify written numerals whose names were given. However, we found no relationship between this rote learning and functional use of this ability until a specific conceptual level was reached. In observing how difficult it is to memorize when the conceptual base for utilization is lacking, and how much easier the memorization is gained when its functional use is immediate and obvious, we recommend that teaching the match between the written numerals and their names not take place until the conceptual skill level necessary for functional use is achieved.

The Perceptual Correspondence Program is designed to teach the hierarchy of perceptual correspondence skills. These perceptual skills are a necessary prerequisite to further mathematical functioning. Without them, students will not progress beyond the rote level of numerical skills. They will not be able to generalize or functionally use their rote numerical skills. Ironically, the Numerical Correspondence Program must then help students move beyond perceptual correspondence and toward conservation of amount. The persistent tug of the perceptual approach long after students have mastered many beginning numerical correspondence skills is testimony to the strength of the perceptual mode and to the necessity of not making assumptions based on developing numerical skills. For example, a student who has mastered the entire hierarchy of counting skills may still, when given a numerical matching task using groups of only four and five objects, match incorrectly based on like configuration rather than on amount.

Earlier it was noted that language skills interface with mathematics skills. In addition to the division of the curriculum into two programs based on presence or absence of expressive language, it was necessary to choose language very carefully in order to avoid confusing lack of language skills with lack of mathematic skills. Five specific areas are described below.

One language-related concern was the "acquiescence syndrome" noted by Sigelman, Budd, Spanel, and Schoenrock (1981). When we asked a question requiring a yes or no response and then reversed the question, many subjects would respond in the affirmative to both. For example, if two trays each had clearly differing amounts of objects, and the examiner pointed to one and asked if it had more, the student would respond, "Yes." The examiner would then repeat the question, pointing to the other tray. The student would again respond in the affirmative. Yet this same student would correctly hand the examiner the tray, bowl, glass or other container holding more (of a choice of two identical containers) using a wide variety of objects and situations.

For this reason, no assessment items require a yes or no response.

We learned that the request, "Show me ____," led to language-based errors rather than measurement of lack of a mathematics skill. For this reason, assessment items request the student to do something, i.e., "Give me the tray with more," rather than, "Show me the tray with more." It may be important to teach students to respond correctly to the request, "Show me ____," but lack of this skill should not prejudice progress in learning mathematics skills. Most if not all functional mathematics skills require action rather than identification.

Students who did not respond to requests using the word "amount" did respond correctly to the same requests when the word "number" was used. Thus, combining our knowledge of the acquiescence syndrome with this piece of information, we asked of two trays of items, "Are they the same number or different number?"

An understandable difficulty but not an obvious one is that relating to the word "one." This word can mean an amount or it can be used as a pronoun as it was in the previous sentence following the word "obvious." When we used the word "one" as a pronoun, as in, "Get enough straws so that every glass has one," the student brought one only. Thus, we avoided this use of "one," repeating the noun instead.

And finally, we found that the concept, "enough," was difficult for many students. Adding the phrase, "No extras," to the request in the previous paragraph results in, "Get enough straws so that every glass has a straw; no extras," and brought the desired response from people at or past that cognitive level. The phrase, "no extras," also helped counter the "no-stop syndrome" described earlier.

Piaget has provided not only a theoretical basis for investigating the developmental order of conceptual mathematics skills in retarded persons but also a basis for devising effective instructional methods based on the student's current conceptual level. For example, because of a characteristic which Piaget called "static thought," persons in the sensori-motor stage tend to focus on the state of an object and to ignore the action transaction that precedes or is performed on the object by another person. Therefore, a corrective action on the part of the teacher will not be comprehended by the student, and he will persist in practicing an incorrect performance. Thus, the student puts an extra object into the container, the common response of the teacher is to remove it with a verbal admonishment, and the student continues to randomly insert extra objects. The teacher's act of removing the object is not focused on by the student nor perceived as a corrective instruction. The student is, in fact, conceptually incapable of comprehending this. In order for the student to learn, it is necessary for the teacher to remove the objects involved in the incorrect response, present a new, identical set, have the student start the activity again with the teacher providing careful cueing so that the correct actions are performed. The student in the sensori-motor stage can not identify the teacher's actions with the resultant object placement but can perceive and remember the desired actions when he performs the activity correctly himself.

As the child moves through the preoperational stage toward the period of logical operations, there are four major characteristics of behavior and thought which must be kept in mind as instruction is planned. The first characteristic, called egocentrism, is the child's conceptual inability to realize other points of view than his own. He operates as if everyone sees and understands things exactly as he does. He doesn't question his own approach

to problem-solving. Therefore, he cannot be taught through our logical explanations. A second characteristic of the thinking of the child in the preoperational stage is his focus on the perceptual elements of a sequence rather than on the transformation taking place in the sequence from a beginning to an ending event. He cannot see the logic inherent in the sequence nor can he understand relationships between events. The third characteristic, centration, refers to the tendency of the child to center or fix his attention on the most outstanding perceptual feature of an event and to ignore the rest. For example, he will focus his perception on the relative sizes of two groups of objects rather than on the amount of objects in each group, even though he knows how to count. The fourth characteristic is the inability to reverse operations. For example, after the child counts identical amounts of objects onto each of two trays, if another person pushes one group into a heap and spreads the other around the outside edges, the child will insist that there are now different amounts of objects.

A dominant element running through all of these characteristics is the child's focus on his own perceptions to the exclusion of all other information and to the exclusion of what we would call logic. The preoperational child's guiding theory might be described as "seeing is believing." This focus on perception means that the preoperational child is not able to conserve. Conservation is the conceptualization or understanding that the amount of quantity of matter or of a group of objects stays the same regardless of changes in shape, position, or arrangement. The student must be confident that a set of objects remains constant while it is counted, separated into groups, or hidden before she can begin to conceptualize and functionally use an even more abstract system, money, and the relative values of coins and bills. When a child reaches this level of understanding, she has advanced from preoperational thought to the stage of concrete operations. (Under conditions of

normal development, this typically happens around the age of six to seven years.)

Functional use of mathematics requires both work and organizational skills as well as basic mathematical concepts. Therefore, a complete curriculum must also teach task completion, organizing materials for greater likelihood of accuracy, checking for accuracy and making necessary corrections, and strategies for performing tasks that would otherwise require mathematics skills beyond the student's level.

The most effective teaching of a prepared curriculum occurs when the instructor understands the specific skill being taught at each step. With this in mind, a curriculum should include an explanation of each skill level to be taught.

We have developed specific teaching techniques for each skill level which use generalized materials and examples in describing how to help the student acquire a new skill. However, it is the teacher's responsibility to apply the idea and process of these teaching techniques to functional tasks and materials in the students' environments. (Suggestions for these tasks are provided, but teachers must adapt them.) In some cases, teachers may be able to completely teach the skill through the use of functional tasks. The more one teaches skills through functional tasks, the better students will learn and generalize. But to teach skills rather than to provide only exposure, the teacher must understand what skill learning she or he is striving for.

Games developed according to skill level are especially useful for providing practice in precision and accuracy. Some games also develop

speed. Games are an excellent teaching medium because they spark enthusiasm and increased effort.

One qualitative feature of our research was to identify differences between autistic students and students of approximately equal overall functioning level who are designated retarded but not autistic. We found three differences, none of them unexpected.

The first difference involves the ritualistic behaviors seen in many autistic students. In order to teach effectively it was necessary to be patient and to work around these behaviors. Non-autistic but retarded students seldom have such behavior. When they do, it is almost always quite easy to shape such behaviors out of their repertoire. The second difference involved social skills. Higher functioning as well as lower functioning autistic students with whom we worked often showed inadequate social skills compared to trainable students. These ranged from severe temper tantrums to non-compliance. The more severely retarded but not autistic students were also subject to such behavior, however, this was not true of the trainable students. The third difference involved language. Our program very carefully follows a normal developmental sequence. Receptive and expressive language become more important as one moves from the lower to the higher level skills. The autistic students as a group evidenced more difficulty with those skills where language concepts were involved than did the retarded students, who followed the developmental sequence much more evenly.

Subsequent to development of this mathematics program, one of the authors became responsible for teaching early elementary students with mild retardation and learning disabilities. The arithmetic materials and workbooks being used

by non-handicapped students were not producing correct and independent work from these handicapped youngsters. When explanations and help were offered, the children tried to learn to do the work by rote. It was quite clear that they were not learning concepts. When assessed using the mathematics program, these children were not successful on one or more of the skills in the Perceptual Correspondence Scale. Thus, they did not yet have the necessary conceptual base for learning numerical skills. The most common difficulty was with the skill which Piaget calls "one to one alignment." Six or more identical objects are placed in a row with the same amount of space between each object. The student is given a container of identical objects and asked to make a row exactly the same as the one laid out. These students made rows in which all of the spaces were filled, thus using many more objects than the model. Until the student sees each object in the original row as discrete and learns to match object for object, he will not be successful in object counting.

Even after one-to-one alignment is mastered, the student may still be tied to the perceptual structure. In this case, perceptual formations will directly influence or alter his numerical counting. If he is asked to place a specific, small number of objects in a row, for example, six objects, he will often miscount or add extras. If he should happen to put the objects down in a circular or haphazard design, he will even more frequently lose count or recount an object. For this reason, counting in specific amounts must be taught in an exact pattern array and in a methodical manner. These youngsters are more familiar with numerals than are the older retarded and autistic students. They responded very well to a combination of checking cards and a counting rhythm. (Checking cards each have a numeral with large, colored dots equal in number to the numeral and placed at strategic places on it.) These provide a pattern formation and placement procedure

for each amount, thus eliminating perceptual confusion and inaccurate object counting. After extensive use of this instructional procedure, the students had learned to count accurately under a wide variety of situations and without these aids. They learned to impose order on the objects with which they were working rather than responding to the objects perceptually.

The use of checking cards and counting rhythms has also proved to be an excellent remedial technique for those students who have mastered most of the numerical amounts to ten but who still have counting difficulty and inaccuracy with a few specific amounts. In this case it is not necessary to reteach all numerical amounts using patterns. The student needs a training procedure only for those amounts he has not accurately mastered.

These mildly retarded and learning disabled youngsters proceeded through the skills of the Numerical Correspondence Scale much more rapidly than the older and more retarded students. Most were eventually able to use the mathematics materials and workbooks used by the rest of the class although they worked more slowly and did not "catch up." These youngsters did not need our entire mathematics program, but parts of it proved crucial to their movement from perceptual correspondence to numerical correspondence and to their development of orderly work habits required for good mathematical performance.

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